MULTIMEDIA UNIVERSITY

STUDENT ID NO

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MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2017/2018

EMT1026 – ENGINEERING MATHEMATICS II
(All Sections / Groups)

5 MARCH 2018

2.30 PM – 4.30 PM

(2 Hours)

INSTRUCTIONS TO STUDENTS:

1. This exam paper consists of **10 pages** (including cover page) with **4** Questions only.
2. Attempt **all** 4 questions. All questions carry equal marks and the distribution of marks for each question is given.
3. Please print all your answers in the Answer Booklet provided. Show all relevant steps to obtain maximum marks.
4. Several tables are provided in the **Appendix** for your reference.

Question 1

(a) (i) Find the **general solution** of the following differential equation:

$$y'' - 4y' - 12y = 3e^{5x}$$

[9 marks]

(ii) Using the above complementary function, find the appropriate form of the particular integral y_p for the differential equation below **(Do not solve it).**

$$y'' - 4y' - 12y = e^{6x} + \sin 6x$$

[3 marks]

(b) Using the power series method, find the first six non-zero terms of the solution for the following differential equation:

$$(1-x)y'' + y = 0$$

[13 marks]

Question 2

The temperature $u(x,t)$ in a laterally-insulated thin metal rod of length L is governed by the heat equation

$$u_t = ku_{xx}, \quad 0 \le x \le L, \quad t > 0,$$

where k is a positive constant. Both ends of the rod are kept at temperature zero for all t.

(a) Show that the general solution for the temperature distribution is

$$u(x,t) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{L}x\right) e^{-\left(\frac{n\pi}{L}\right)^2 kt}.$$

[18 marks]

(b) Find the specific solution $u(x,t)$ if the initial temperature across the rod is $u(x,0) = x$. (The formulae in APPENDIX A may be useful for solving this question.)

[7 marks]

Continued...

Question 3

(**Note**: The tables in APPENDIX B and APPENDIX C may be useful for solving this question.)

(a) Let $f(t)=e^{-2t}$ and $g(t)=t\cos 2t$.

(i) Find the Laplace transform of $g(t)$. [4 marks]

(ii) Hence, show that $L\{f(t)*g(t)\}=\dfrac{s-2}{(s^2+4)^2}$, where $*$ denotes convolution.

[4 marks]

(b) An s-domain function is defined by

$$H(s)=\frac{1}{2s(s-1)}.$$

(i) Determine the inverse Laplace transform of $H(s)$. [4 marks]

(ii) Rework part (i), but using a *different* transform property. Show that your final answer agrees with that from part (i). [4 marks]

(c) Consider the following ordinary differential equation:

$$x''(t)-x'(t)-2x(t)=0, \text{ subject to } x(0)=1 \text{ and } x'(0)=0.$$

By applying the Laplace transform, show that

$$L\{x(t)\}=X(s)=\frac{s}{(s-2)(s+1)}.$$

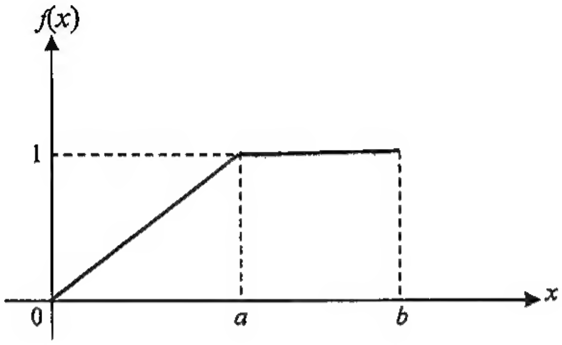
Hence, obtain the complete solution of the above differential equation.

[9 marks]

Continued...

Question 4

(a) The probability density function $f(x)$ of a random variable X is graphed below:



(i) Show that $b-\dfrac{a}{2}=1$. [2 marks]

(ii) Given also that $P(X<a)=P(X>a)$, use this information together with part (i) to deduce that $a=1$ and $b=3/2$. [4 marks]

(iii) Develop a piecewise algebraic expression for $f(x)$. Hence, evaluate the mean of X. [4 marks]

(**Note**: For solving Q4(b) and Q4(c) below, you may use the tables in APPENDIX D, APPENDIX E and APPENDIX F.)

(b) Eggs from a chicken farm are packed in cartons of 10 eggs/carton to be shipped to nearby hypermarkets. On average, 1% of the eggs crack during shipment.

(i) Find the probability that a carton will contain more than 1 cracked egg. [4 marks]

(ii) A hypermarket receives a shipment of 500 cartons of eggs. How many of these cartons will have no cracked eggs? [3 marks]

(c) A cargo parachute has been designed to automatically open at an average height of $\mu=200$ m above ground. Due to mechanical imprecision, the actual opening altitude is normally distributed with a standard deviation of $\sigma=20$ m.

(i) Find the probability that the parachute opens when it is between 175m to 225m from the ground. [4 marks]

(ii) Cargo damage will occur if the parachute opens at an altitude of less than 150m. Out of 500 parachute drops, how many will probably **not** result in damaged cargo? [4 marks]

End of questions.

APPENDIX A

Half-range sine series:

$$f(x)=\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right), \quad \text{where } b_n=\frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

Half-range cosine series:

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right), \quad \text{where } a_0=\frac{2}{L}\int_0^L f(x)dx, \ a_n=\frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$$

Continued...

APPENDIX B:

Common Laplace Transform Pairs

	$f(t)$	$F(s) = L\{f(t)\}$
1.	1	$\frac{1}{s}$
2.	t	$\frac{1}{s^2}$
3.	t^n	$\frac{n!}{s^{n+1}}, \ n = 1,2,\ldots$
4.	e^{at}	$\frac{1}{s-a}$
5.	$t^{n-1}e^{at}$	$\frac{(n-1)!}{(s-a)^n}, \ n = 1,2,\ldots$
6.	$\cos at$	$\frac{s}{s^2+a^2}$
7.	$\sin at$	$\frac{a}{s^2+a^2}$
8.	$\cosh at$	$\frac{s}{s^2-a^2}$
9.	$\sinh at$	$\frac{a}{s^2-a^2}$
10.	$u(t-a)$	$\frac{e^{-as}}{s}$
11.	$\delta(t-a)$	e^{-as}

Continued...

APPENDIX C:

Laplace Transform Properties

	Property Name	Formula
1.	Linearity	$L\{af(t)+bg(t)\}=aL\{f(t)\}+bL\{g(t)\}$
2.	s - shifting	$L\{e^{at}f(t)\}=F(s-a)$
3.	Transform of Derivative	$L\{f'(t)\}=sL\{f(t)\}-f(0)$ $L\{f''(t)\}=s^2L\{f(t)\}-sf(0)-f'(0)$
4.	Transform of Integration	$L\left\{\int_0^t f(\tau)d\tau\right\}=\frac{1}{s}F(s)$
5.	t - shifting	$L\{u(t-a)f(t-a)\}=e^{-as}F(s)$
6.	Differentiation of Transform	$L\{t.f(t)\}=-F'(s)$
7.	Integration of Transform	$L\left\{\frac{f(t)}{t}\right\}=\int_s^\infty F(\tilde{s})d\tilde{s}$
8.	Convolution Theorem	$L\{f(t)*g(t)\}=F(s).G(s)$, where $f(t)*g(t)=\int_0^t f(\tau)g(t-\tau)d\tau$.

Continued...

APPENDIX D:

Special Discrete Probability Distributions

Binomial Distribution, $X \sim b(x; n, p)$	
P.m.f.	$P(X = x) = {}^{n}C_{x} p^{x} q^{n-x}$, for $x = 0, 1, \ldots, n$ and where $q = 1 - p$.
Mean	$E[X] = np$
Variance	$Var(X) = npq$
Hypergeometric Distribution, $X \sim h(x; N, n, k)$	
P.m.f.	$P(X = x) = \dfrac{{}^{k}C_{x} \times {}^{N-k}C_{n-x}}{{}^{N}C_{n}}$, for $x = 0, 1, \ldots \min(n, k)$.
Mean	$E[X] = \dfrac{nk}{N}$
Variance	$Var(X) = \dfrac{nk}{N}\left(\dfrac{N-n}{N-1}\right)\left(1 - \dfrac{k}{N}\right)$
Poisson Distribution, $X \sim p(x; \lambda)$	
P.m.f.	$P(X = x) = \dfrac{\lambda^{x} e^{-\lambda}}{x!}$, for $x = 0, 1, \ldots$.
Mean	$E[X] = \lambda$
Variance	$Var(X) = \lambda$

Continued...

APPENDIX E:

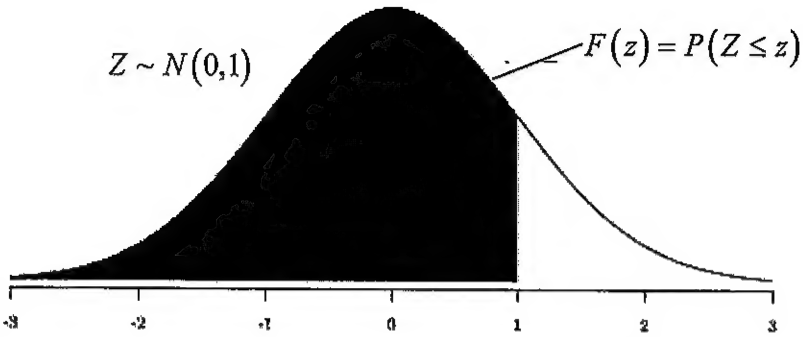
Special Continuous Probability Distributions

Continuous Uniform Distribution, $X \sim U(a,b)$	
P.d.f.	$f_X(x)=\begin{cases}\dfrac{1}{b-a}, & a<x<b\\ 0, & \text{elsewhere}\end{cases}$
Mean	$E[X]=\dfrac{a+b}{2}$
Variance	$Var(X)=\dfrac{(b-a)^2}{12}$
Exponential Distribution, $X \sim \exp(1/\beta)$	
P.d.f.	$f_X(x)=\begin{cases}\dfrac{1}{\beta}e^{-x/\beta}, & x>0\\ 0, & \text{elsewhere}\end{cases}$
Mean	$E[X]=\beta$
Variance	$Var(X)=\beta^2$
Normal Distribution, $X \sim N(\mu,\sigma)$	
If X is any normal random variable where $X \sim N(\mu,\sigma)$, then the transformation $$Z=\frac{X-\mu}{\sigma}$$ yields a *standard* normal variable where $Z \sim N(0,1)$.	

Continued...

APPENDIX F:

Cumulative Standard Normal Distribution



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

End of paper.